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Problem-Solving Errors in Highest Common Factors and Lowest Common Multiples: An Analysis of Pre-Service Teachers' Misconceptions

Shakespear Maliketi Chiphambo
Mathematics, Science and Technology Education Department, Faculty of Education
Walter Sisulu University, South Africa, schiphambo@wsu.ac.za

ABSTRACT

This study examined the conceptual understanding of South African pre-service teachers regarding the Highest Common Factor (HCF) and Lowest Common Multiple (LCM), with a focus on errors and misconceptions that affect problem-solving. There is a paucity of research that specifically analyses how pre-service teachers' misconceptions about HCF and LCM contribute to problem-solving errors in foundational mathematics. This study is grounded in two complementary theoretical perspectives: the Mathematical Knowledge for Teaching (MKT) framework and the Conceptual–Procedural Knowledge Theory. A sequential explanatory mixed-methods design was employed. Quantitative data were analysed using one-sample t-tests and Pearson's correlation, and qualitative data were supported by error analysis. The three hypotheses were tested. Results showed a significant difference in understanding ($t = 25.685$, $p < 0.001$), leading to the rejection of the first hypothesis. A moderate negative correlation ($r = -0.48$, $p < 0.001$) between misconceptions and accuracy led to the rejection of the second hypothesis. The third hypothesis was not rejected, as no strong correlation was found between strategy use and conceptual errors ($r = -0.12$, $p = 0.425$). Findings highlight gaps in conceptual understanding and emphasise the need for conceptually focused instruction, diagnostic assessments, and real-life applications in teacher education programmes.

Keywords: Common Factors, Common Multiples, Pre-service teachers

ABSTRAK

Studi ini menguji pemahaman konseptual calon guru Afrika Selatan mengenai Faktor Persekutuan Tertinggi (FPG) dan Kelipatan Persekutuan Terendah (KPK), dengan fokus pada kesalahan dan kesalahpahaman yang memengaruhi pemecahan masalah. Ada kekurangan penelitian yang secara khusus menganalisis bagaimana kesalahpahaman calon guru tentang FPB dan KPK berkontribusi pada kesalahan pemecahan masalah dalam matematika dasar. Studi ini didasarkan pada dua perspektif teoretis yang saling melengkapi: kerangka Pengetahuan Matematika untuk Mengajar (MKT) dan Teori Pengetahuan Konseptual-Prosedural. Desain metode campuran penjelasan sekuensial digunakan. Data kuantitatif dianalisis menggunakan uji-t satu sampel dan korelasi Pearson, dan data kualitatif didukung oleh analisis kesalahan. Ketiga hipotesis diuji. Hasil menunjukkan perbedaan yang signifikan dalam pemahaman ($t = 25{,}685$, $p < 0{,}001$), yang mengarah pada penolakan hipotesis pertama. Korelasi negatif sedang ($r = -0{,}48$, $p < 0{,}001$) antara miskonsepsi dan akurasi menyebabkan penolakan hipotesis kedua. Hipotesis ketiga tidak ditolak, karena tidak ditemukan korelasi kuat antara penggunaan strategi dan kesalahan konseptual ($r = -0{,}12$, $p = 0{,}425$). Temuan ini menyoroti kesenjangan dalam pemahaman konseptual dan menekankan perlunya instruksi yang berfokus pada konsep, asesmen diagnostik, dan aplikasi kehidupan nyata dalam program pendidikan guru.

Kata kunci: Faktor Persekutuan, Kelipatan Persekutuan, Calon Guru

INTRODUCTION

In mathematics, the concepts of Highest Common Factor (HCF) and Lowest Common Multiple (LCM) serve as foundational tools for understanding the relationships between numbers and operations. The curriculum focuses on building understanding, yet pre-service teachers exhibit gaps in their understanding of HCF and LCM (Lumadi, 2014). Studies show that pre-service teachers often face challenges with underlying concepts, such as the HCF and LCM, although they seem to acquire a certain level of procedural fluency (Mbhiza, 2024). For example, this lack of core knowledge seems to pervade many educational programmes, contributing to the complex problem of mathematics teaching in South Africa (Saal & Graham, 2023). Furthermore, there is a need to assess the integration of basic conceptual knowledge into the curriculum for pre-service teacher educators, considering the severe reality of unaddressed gaps in basic concepts their trainees have upon entering teaching (Makonye, 2017).

Understanding the obstacles that pre-service teachers encounter when learning HCF and LCM is important (Motilal & Fleisch, 2020). Improving the understanding of these basic educational building blocks has the potential to enhance the quality of mathematics instruction, which is crucial for student performance (Taley, 2022). The national concern about poor performance in mathematics places a greater responsibility to equip students with sound primary mathematics knowledge necessary for effective teaching (Joubert & Kenny, 2018).

This study contributes by providing one of the few empirical analyses that directly link pre-service teachers' misconceptions of HCF and LCM to specific problem-solving errors using an integrated framework that combines the Mathematical Knowledge for Teaching (MKT) model with the Conceptual–Procedural Knowledge Theory. Unlike previous studies that examine number concepts in isolation, this research identifies how conceptual and procedural weaknesses manifest within foundational mathematical tasks. It further demonstrates how these weaknesses correspond to specific components of teacher knowledge required for effective instruction. The mixed-methods design further strengthens the contribution by not only quantifying the extent of misconceptions but also qualitatively explaining the nature and origins of those errors. Through this approach, the study offers novel insights into the diagnostic patterns of error in pre-service teacher learning and provides evidence-based implications for strengthening mathematics teacher education programmes.

Objectives

The study aimed to:

1. Explore the conceptual understanding of HCF and LCM among pre-service teachers.
2. Investigate the common errors or misconceptions that pre-service teachers exhibit when solving HCF and LCM problems.

Research Questions

The research sought to answer the following questions:

1. What is the level of conceptual understanding of HCF and LCM among second-year pre-service teachers in South Africa?

2. What common errors or misconceptions do pre-service teachers exhibit when solving HCF and LCM problems?

Hypotheses

The following hypotheses were tested at a 0.05 significance level:

1. H_{01}: There is no significant difference in pre-service teachers' conceptual understanding of HCF and LCM.
2. H_{02}: Pre-service teachers' misconceptions about HCF and LCM do not significantly affect their problem-solving accuracy.
3. H_{03}: The use of incorrect strategies is not significantly correlated with conceptual errors in HCF and LCM questions.

By illuminating these issues, the research contributes insights that enhance the effectiveness of teacher education programs and ultimately improve mathematics education outcomes across South African schools.

THEORETICAL FRAMEWORK

This study is grounded in two complementary theoretical perspectives: the Mathematical Knowledge for Teaching (MKT) framework and the Conceptual–Procedural Knowledge Theory. The MKT framework, developed by Ball, Thames, and Phelps (2008), provides a comprehensive lens for understanding the specialised mathematical and pedagogical knowledge required for effective teaching. It highlights how teachers must perform mathematical procedures and explain underlying concepts, anticipate learner errors, and select appropriate representations. Complementing this, the Conceptual-Procedural Knowledge Theory, articulated by Hiebert and Lefevre (1986) and underpinned by Skemp's (1976) earlier distinction between relational and instrumental understanding, distinguishes between deep conceptual understanding and procedural fluency in mathematical problem-solving. Integrating these theories strengthens the analytical power of this study by linking the nature of pre-service teachers' HCF and LCM errors to specific components of teacher knowledge: conceptual errors signal weaknesses in the conceptual foundation central to MKT's specialised content knowledge, while procedural errors reveal gaps in the procedural fluency and instructional decisions associated with MKT's pedagogical domains. Together, these theories form a coherent framework for examining how misconceptions disrupt problem-solving and for identifying the types of knowledge teacher education programmes must develop.

LITERATURE REVIEW

Mathematics understanding of conceptual and procedural knowledge

Different types of knowledge in mathematics education stem from the fundamental understanding that students possess, and the two dimensions necessary in their education are conceptual and procedural knowledge. Herheim (2023) notes that relational understanding correlates somewhat with conceptual understanding, whereas instrumental understanding, which is of a lower hierarchy than relational understanding, refers to a basic level of understanding of the techniques. As with the abstract notion of mathematics, understanding and application of mathematics also require foundational skills such as memorisation.

Understanding concepts in mathematics requires students to internalize algorithms accordingly (Benson et al., 2023). For example, students often use the order of operations to perform calculations without thinking about their meaning. In other words, when performing operations on numerical sets, students strive to internalise steps so that they can automatically execute their procedures. Each component of math has defined parameters to guide the problem-solving techniques users develop. Alternatively, each problem arises from within its own set of defining parameters, aiming to solve that specific issue (Herheim, 2023).

The significance of developing conceptual understanding in teaching certain mathematical topics, such as the Highest Common Factor (HCF) and Lowest Common Multiple (LCM), cannot be overstated. With proper conceptual understanding, students know when to apply HCF as opposed to LCM, which then enhances their problem-solving skills and real-life applications (Halim et al., 2017). Theoretical frameworks, especially Ball et al.'s (2008) theory on relational versus instrumental understanding, help highlight this gap. Relational understanding is instrumental in enabling students to derive meaning and identify relationships, thereby improving their effectiveness in solving various mathematical problems (Herheim, 2023) and ensuring the development of a strong mathematical identity and self-efficacy in mathematics (Yang et al., 2021).

Teaching and Learning HCF and LCM

As with most curricular expectations related to the teaching of HCF and LCM, both procedural fluency and understanding are included. Most curricula, for example, Curriculum Assessment Policy Statements (CAPS) and others, are concerned with a systematic structure that facilitates teaching and learning, thereby enhancing understanding, as seen in CAPS (Halim et al., 2017). Instructional strategies used include listing factors, Venn diagrams, and prime factorisation as approaches to HCF and LCM problems. Such strategies not only develop performance skills but also seek to create proper connections between concepts (Halim et al., 2017; Caniglia & Meadows, 2018).

Regardless of the steps used to teach, students continue to struggle with the misconceptions related to HCF and LCM. Some students can mix these concepts and apply them inappropriately through rote memory, without a clear understanding (Halim et al., 2017). Such errors are made most of the time due to ignorance, which emanates from a lack of understanding of the distinguishing features of concepts that separate HCF from LCM in one's procedural knowledge. These are errors that can be remedied by unlearning rote strategies and developing model strategies that facilitate flexible thinking frameworks (Semper & Lizasoain, 2023).

Number theory concepts are erroneously assumed to be correct

It has been demonstrated that misconceptions about fundamental concepts in number theory, such as HCF and LCM, persist among students and pre-service teachers alike (Halim et al., 2017). These misconceptions typically stem from gaps in learning that have accumulated over the preceding years of schooling. This highlights the need for prior knowledge to be considered in the classroom.

For example, students encounter challenges with the abstract ideas of number theory due to a lack of conceptual scaffolds developed earlier in their education (Benson et al., 2023).

Furthermore, literature on error analysis in mathematics demonstrates that the misconceptions underlying the errors frequently stem from a lack of relational understanding (Yang et al., 2021). Therefore, the interaction between gaps in fundamental understanding and students' difficulties with HCF and LCM highlights the importance of integrated strategies designed to address these gaps while providing clarity on basic concepts (Furner, 2018; Caniglia & Meadows, 2018). With awareness of common misconceptions, dedicated strategies can be designed that target procedural and conceptual gaps to create robust frameworks of understanding and appreciation for fundamental mathematical concepts (Semper & Lizasoain, 2023; Yang et al., 2021).

Pre-Service Teachers' Mathematical Knowledge for Teaching (MKT)

The role of pre-service teachers is crucial in the educational ecosystem, particularly in mathematics, as their knowledge has a significant impact on students' results. Research indicates that many PSTs start their training with inadequate mathematical foundations, which hinders their development of effective didactic strategies (Mosvold, 2022; Chikiwa & Graven, 2023). The Mathematical Knowledge for Teaching (MKT) framework issued by Ball, Thames and Phelps (2008) identifies two primary components: content knowledge and pedagogical content knowledge (PCK) (Mosvold, 2022). Content knowledge refers to the fundamental concepts and processes that teachers possess regarding mathematics taught in schools. PCK refers to how content is taught, in addition to students' perceptions of diverse mathematical processes (Mosvold, 2022).

Chikiwa & Graven (2023) found that gaps in content knowledge and PCK among South African PSTs limit their self-efficacy and preparedness to teach mathematics. Enhancing MKT among PSTs needs well-developed teacher education programmes that are relevant to the context and support the application of mathematical concepts toward improving PCK and mathematical literacy in the students (Chikiwa & Graven, 2023). Moreover, incorporating reflection as a component of teaching practices can enable the development of MKT in PSTs, allowing them to respond appropriately to classroom dynamics (Chikiwa & Graven, 2023). Therefore, it appears that South African PSTs require an integrated approach that balances content and pedagogy to satisfy the requirements of effective mathematics teaching.

Real-life impacts of HCF and LCM teaching

The context of enhancing understanding and relevance among students has shown that teaching mathematics with relevance provides meaning within the classroom setting. The concepts of Highest Common Factor (HCF) and Lowest Common Multiple (LCM) can be best learned through practical, real-life cases, ensuring that these abstract mathematical concepts are accessible (Chikiwa & Graven, 2023). For instance, scheduling and distributing items evenly or word problems incorporating such real-life settings illustrate the usefulness of the HCF and LCM intersection in real life. This not only fosters a deep understanding of concepts but also the application of mathematics, hence motivating students (Chikiwa & Graven, 2023).

Application problems bridge the gap between theoretical knowledge and practical skills, revealing students' understanding and practical application of mathematical concepts (Chikiwa &

Graven, 2023). Inasmuch as teachers facilitate discussion on real-life concerns, this opens the window of creativity for students to think of diverse strategies to explain their approach to the problem. This fosters critical and divergent thinking, prompting PSTs to rethink how to plan lessons that highlight the relational nature of the concept and improve their own MKT in strategies designed for teaching future students (Chikiwa & Graven, 2023).

Gaps in South African mathematics teacher education

South Africa's mathematics teacher education system is a result of a serious systemic problem, which entails a lack of sufficient teacher preparation, low student mathematical skills, and poor curriculum implementation at the school level. Studies show that teachers often fail to adequately grasp the concepts that need to be taught at a sophisticated level, such as in a primary school Euclidean geometry class (Tachie, 2020). The teaching practice component does not appear to adequately equip prospective teachers with the skills necessary to deliver quality teaching (Jojo, 2020). In addition, the COVID-19 pandemic exacerbated pre-existing inequalities, as the sudden shift to online learning highlighted numerous gaps in technological and resource availability across different areas (Chirinda et al., 2021). Their inability to exercise effective teaching strategies profoundly influenced educational outcomes (Landa et al., 2021).

Research discloses a worrying trend where educational policies enacted after apartheid do not adequately address the intersectional disparities within the South African education system (Wiseman & Davidson, 2021). It underscores the need to address community engagement processes where policies developed do not integrate the complex realities within the educational ecosystem (Wiseman & Davidson, 2021). Nonetheless, there is little attention given to the actual teaching practices that foster this equity, which stems from an overemphasis on pedagogy in the teacher education curriculum rather than the implementation of equitable teaching practices (McDonald et al., 2021).

In a country with as many languages as South Africa, language, along with prior knowledge, plays a critical part in mathematics education. Evidence suggests that a significant number of students struggle with mathematics due to language barriers, which can hinder their understanding of the concepts and terminology associated with the subject (Chirinda et al., 2021). This problem is worsened by the fact that schools in previously disadvantaged areas tend to be under-resourced and inadequately staffed to provide quality education (Ngobeni et al., 2023).

Furthermore, the prior knowledge that students bring into their learning greatly affects their ability to interact with new concepts, especially in mathematics (Mbhiza, 2024). These differences in socio-economic contexts among students also bring out the educational inequalities in students' experiences. To address these, relevant policies are required to bridge the differences in the education systems (Sadiki et al., 2023; Ngobeni et al., 2023).

One plausible explanation for the limited development of mathematics education is the persistent neglect of social injustices within curricular and instructional frameworks. Viewing mathematics through a social justice lens exposes unimplemented policies that hinder coordinated, systemic reform. This perspective emphasises designing mathematics teaching that engages inequities, stratified social structures, and entrenched societal divisions, enabling learners to critically understand and respond to these conditions. Solving the problem concerning the education of teachers with respect to teaching mathematics needs a broad approach, firstly, to improve teacher professional development and curriculum implementation. Secondly, to address the disparities found in the education system. There is no simple solution, at least not yet, as far as language and prior knowledge differences are concerned. However, collaboration through education policy on addressing those inequities raises opportunities for equitable mathematical education.

METHOD

Research Design

Guided by a pragmatic paradigm, this study employed a sequential explanatory mixed-methods design to investigate pre-service teachers' conceptual understanding of the Highest Common Factor (HCF) and Lowest Common Multiple (LCM). The mixed-methods design process used in this study is illustrated in Figure 1 below. The research began with the collection of quantitative data through a structured class test, based on items adapted from the Annual National Assessment (ANA), followed by a qualitative phase that included a document review on HCF and LCM. The combination of methods enabled triangulation and deeper insight into students' misconceptions. A census sampling technique was used, involving all 60 second-year Bachelor of Education students from a South African university. This approach enabled a comprehensive and contextually rich analysis of both the measurable performance and the experiential challenges faced by the participants (Fetters & Molina-Azorín, 2017).

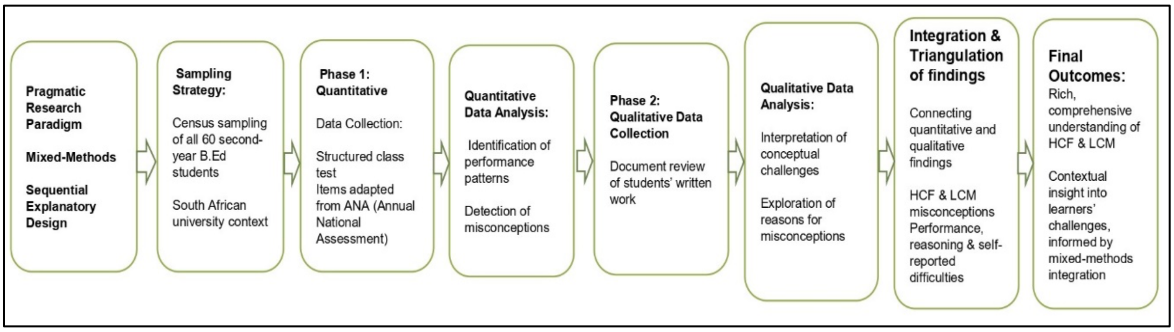

Figure 1. The Flow chart of the mixed methods design

Content Validity

The content validity of the instrument was established through a structured expert-review process. Three subject-matter specialists in mathematics education, all with extensive experience in teaching and curriculum design, independently evaluated the assessment items for alignment with the targeted curricular constructs and the cognitive demands specified in the research framework. Reviewers examined each item for representativeness, clarity, developmental appropriateness, and potential sources of ambiguity. Their feedback informed a round of revisions that refined item wording, adjusted difficulty levels, and ensured balanced coverage of key conceptual domains. This systematic review process provided evidence that the instrument adequately reflects the content and skills it was designed to measure.

Data Analysis

All analyses of the three hypotheses were conducted at an α = 0.05 significance level. The first hypothesis was analysed using a one-sample t-test, and the other two hypotheses were analyzed using Pearson's Correlation Test. Error analysis was used to identify and interpret errors and misconceptions students hold regarding HCF and LCM concepts.

RESULTS AND DISCUSSION

Result

All analyses of the three hypotheses were conducted at an α = 0.05 significance level, with appropriate checks for statistical assumptions and effect size calculations where applicable. The results of the statistical analysis are presented in Table 1.

1. H_0: There is no significant difference in pre-service teachers' conceptual understanding of HCF and LCM. A one-sample t-test was used for this hypothesis.
2. H_0: Pre-service teachers' misconceptions about HCF and LCM do not significantly affect their problem-solving accuracy. The analysis used: Pearson's Correlation Test, focusing on Correlated the misconception score and with problem-solving accuracy score
3. H_0: The use of incorrect strategies is not significantly correlated with conceptual errors in HCF and LCM questions. The Pearson's Correlation Test was used to correlate the strategy effectiveness score with the misconception score.

Table 1. A Summary of Hypotheses, Statistical Tests, and Findings

Hypothesis		Statistical Test	Sig. Level	Result	Interpretation
H_{01}	: No significant difference in PSTs' conceptual understanding of HCF and LCM.	One-sample t-test	α = 0.05	Rejected (p < 0.001, t = 25.685, d = 3.29)	PSTs' mean score significantly exceeded the hypothesised mean, indicating strong conceptual understanding.
H_{02}	: PSTs' misconceptions do not significantly affect problem-solving accuracy.	Pearson's correlation	α = 0.05	Rejected (p < 0.001, r = –0.48)	Higher misconceptions are associated with reduced accuracy in problem-solving.
H_{03}	: Incorrect strategy use is not significantly correlated with conceptual errors.	Pearson's correlation	α = 0.05	Not rejected (p = 0.425, r = –0.12)	Strategy choice shows no significant relationship to misconception levels.

The statistical analysis of the three hypotheses revealed the following:

1. $p < 0.001$, $t = 25.685$, as a result, the first hypothesis, H_0: There is no significant difference in pre-service teachers' conceptual understanding of HCF and LCM, is rejected. The large effect

size (Cohen's d = 3.29) indicates this difference is practically significant. This suggests that the pre-service teachers' mean score was significantly different from the hypothesised mean, indicating a strong conceptual understanding of HCF and LCM. MKT Framework Connection: Common Content Knowledge (CCK): The significant mean score of 4.37 indicates a strong basic mathematical understanding. Specialised Content Knowledge (SCK): The high effect size (d = 3.29) suggests pre-service teachers possess the unique mathematical knowledge needed to unpack HCF and LCM concepts. Knowledge of Content and Students (KCS): Strong conceptual understanding enables teachers to anticipate students' thinking and misconceptions. This agrees with Halim et al.'s (2017) research which revealed that with proper conceptual understanding, students can apply HCF as opposed to LCM, which then enhances their problem-solving skills and real-life applications

2. $p < 0.001$, $r = -0.48$, leads to the rejection of the second hypothesis, H_0: Pre-service teachers' misconceptions about HCF and LCM do not significantly affect their problem-solving accuracy. The negative correlation suggests that higher misconceptions are associated with lower problem-solving accuracy, indicating that as misconceptions increase, problem-solving accuracy tends to decrease. MKT Framework Connection: - Horizon Content Knowledge (HCK): The negative correlation shows how gaps in mathematical knowledge affect teaching capability - Knowledge of Content and Teaching (KCT): The moderate negative correlation ($r = -0.48$) indicates that teachers' misconceptions directly impact their ability to present and sequence HCF/LCM content - Knowledge of Content and Curriculum (KCC): Teachers with higher misconception scores showed reduced ability to connect HCF/LCM to broader mathematical concepts.
3. The results, $p = 0.425$, $r = -0.12$, fail to reject the third hypothesis, H_0: The use of incorrect strategies is not significantly correlated with conceptual errors in HCF and LCM questions. Strategy use scores and misconception scores showed a weak negative correlation, suggesting that the choice of strategy used by pre-service teachers is not significantly related to their level of misconceptions. MKT Framework Connection: - Specialised Content Knowledge (SCK): The non-significant correlation suggests that strategy selection (prime factorisation vs. listing multiples) does not necessarily indicate conceptual understanding - Knowledge of Content and Teaching (KCT): The weak correlation implies that pre-service teachers might use different strategies effectively regardless of their conceptual understanding. Knowledge of Content and Students (KCS): The results suggest that pre-service teachers' choice of strategy isn't strongly linked to their ability to understand student errors.

Question by question analysis

Strong conceptual understanding (Hypothesis 1) suggests that effective basic training in HCF and LCM is necessary. The significant impact of misconceptions (Hypothesis 2) highlights the need for targeted interventions in teacher preparation programs. The lack of correlation between strategy use and errors (Hypothesis 3) suggests the need for a more explicit connection between procedural and conceptual knowledge in teacher education. This confirms Chikiwa and Graven's (2023) study, which revealed that gaps in content knowledge and PCK among South African PSTs limit their self-efficacy and preparedness to teach mathematics. These findings align with Ball et al.'s (2008) emphasis on the multifaceted nature of mathematical knowledge required for teaching, particularly highlighting the interconnection between content knowledge and pedagogical knowledge in teaching HCF and LCM effectively.

A descriptive analysis of the conceptual understanding of HCF and LCM among second-year prospective teachers in South Africa is presented in Table 2. This table clearly presents central tendency, dispersion, and variability for the students' performance.

Table 2. A summary of the overall performance on the HCF and LCM questions.

Descriptive Statistic	Value (%)
Mean Score	61.25
Median Score	62.5
Standard Deviation	35.48
Interquartile Range (Q1–Q3)	25 – 100

Question-specific performance

Question 2.1.3 (Basic HCF concept). This question focuses on assessing students' knowledge and understanding of the HCF, a fundamental concept in mathematics that involves finding the largest number that divides two or more numbers without leaving a remainder. This has the highest success rate of 81.67%, which demonstrates a strong conceptual understanding of basic HCF principles

Question 2.6 was designed to evaluate students' ability to apply the concept of LCM in problem-solving scenarios (Complex LCM application). The success rate was 68.89%, which demonstrates an acceptable performance on advanced LCM applications

Question 2.1.2 (Basic LCM concept). This question was designed to assess students' understanding of the concept of the LCM. The results show a moderate success rate of 56.67%, which reveals gaps in basic LCM understanding

Question 2.5 (Complex HCF application) required students to understand the basic concept of HCF and to apply it in a more challenging scenario. The question involves multiple steps, requires critical thinking, or integrates HCF with other mathematical concepts. The question has the Lowest success rate of 48.33%, suggesting difficulties with the complex HCF application. This finding aligns with Benson et al. 's (2023) study, which identified a lack of conceptual scaffolds developed earlier in students' education as the cause of abstract ideas in number theory.

The different distribution levels of students' proficiency of HCF and LCM were as follows: 47.27% of students scored were at an advanced level of proficiency with a score of 75% and above, 12.73% of the students considered to be at a proficient level scored 50-74%, Basic (30-49%): 18.18% of the students were rated at basic score of 30-39%. Lastly, 21.82% of the students performed poorly, scoring less than 30%.

A document review of the test scripts revealed common errors and misconceptions that pre-service teachers exhibit when solving HCF and LCM problems, as shown in Figure 2.

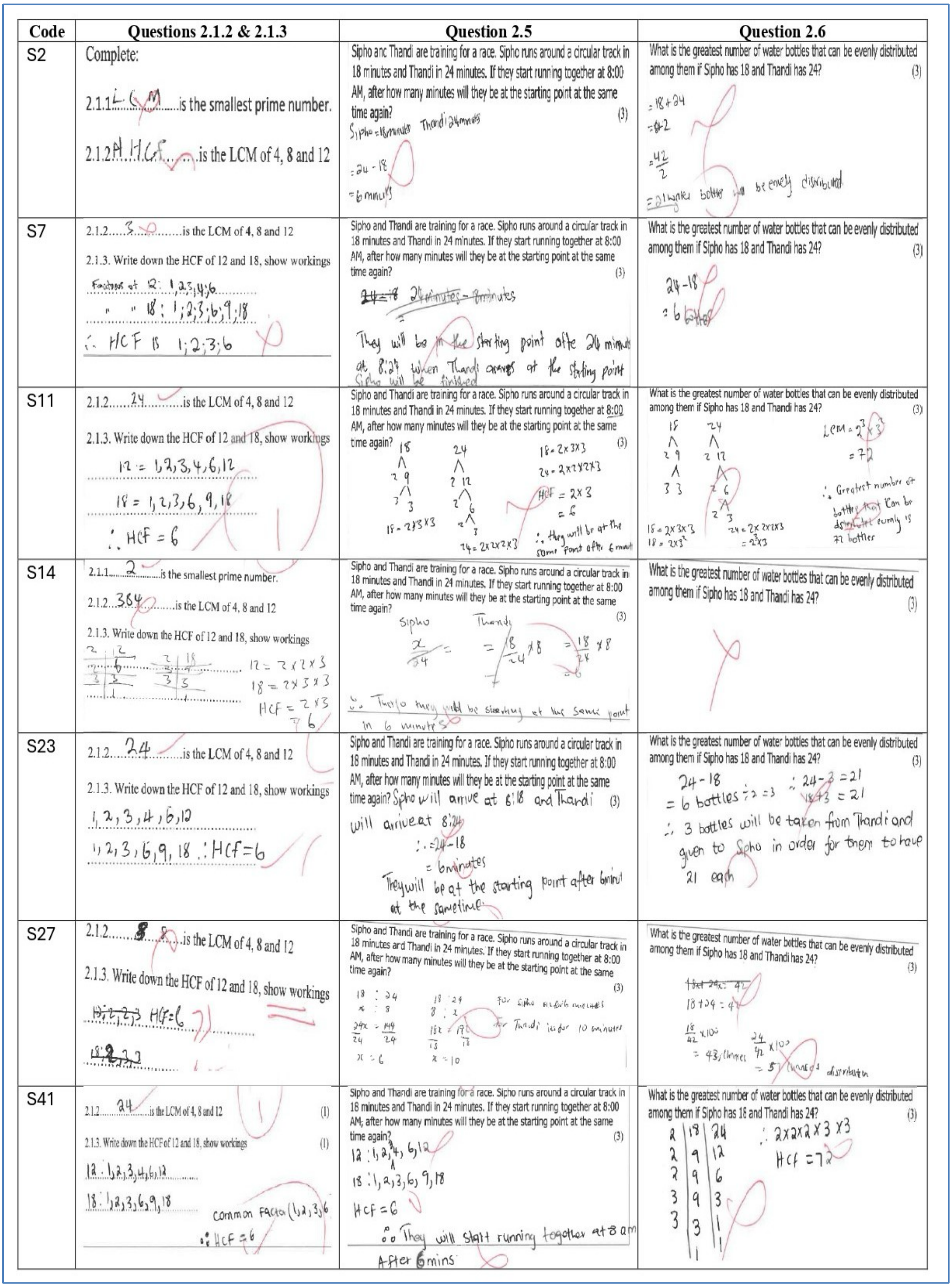

Figure 2. Illustrates the selected misconceptions of pre-service teachers when solving HCF and LCM problems.

Analysis of students' responses to Questions 2.1.2; 2.1.3; 2.5 and 2.6
Question prompt: 2.1.2 ………………… is the LCM of 4, 8, and 12.

Correct Answer:

The Least Common Multiple (LCM) of 4, 8, and 12 is 24.
To calculate it:
Prime factorisation:
$4 = 2^2$
$8 = 2^3$
$12 = 2^2 \times 3$
LCM = product of the highest powers of all prime factors involved
$= 2^3 \times 3 = 24$

As shown in Figure 2, S2 wrote the phrase "said LCM" instead of a numerical answer, indicating a misunderstanding of the question's purpose. This confirms the language difficulties that some students experience, which obstruct their understanding of the concepts and terminology associated with mathematics (Chirinda et al., 2021).

CCK lens: S2 lacks basic procedural knowledge to calculate the LCM. Such a response suggests that either S2 did not understand what was being asked or believed the question was requesting a restatement or definition rather than a computation. This reflects Halim et al.'s (2017) argument, which highlights that such a lack of understanding shows a failure to grasp the distinguishing features of concepts that separate HCF from LCM in one's procedural knowledge.

SCK lens: From a teaching perspective, a lecturer must recognise this response as a semantic or linguistic misunderstanding rather than a purely mathematical error. The lecturer's SCK must help in distinguishing whether the student is struggling with mathematical language (e.g., not understanding what "LCM" refers to) or the concept itself.

Based on Figure 2, S7's response is 3. The number 3 is not a common multiple of 4, 8, and 12; instead, it may reflect confusion with the concept of Greatest Common Factor (GCF).

CCK Lens: S7 confuses factors with multiples, indicating a foundational conceptual misunderstanding. This indicates a weakness in CCK's ability to distinguish between GCF and LCM, two central and commonly taught concepts in elementary number theory. Research suggests that students can mix these concepts and apply them inappropriately through rote memory, without understanding the underlying principles (Halim et al., 2017).

SCK Lens: A lecturer with strong SCK should anticipate this kind of error and design teaching strategies that explicitly contrast GCF and LCM. For example, using Venn diagrams of prime factors or real-life contexts (e.g., arranging items into rows for GCF vs. finding shared event times for LCM).

S14's response is "384", 384 is a common multiple of 4, 8, and 12, but not the least. It is also the product of $4 \times 8 \times 12$, which strongly suggests a procedural error: calculating the product instead of the LCM. This confirms research findings that highlight gaps in the understanding of HCF and LCM among pre-service teachers (Lumadi, 2014).

CCK Lens: S14 demonstrated some understanding that a common multiple is needed, but lacks accuracy in identifying the least one. This reflects a lack of procedural fluency in using efficient algorithms, such as prime factorisation or listing multiples.

SCK Lens: The lecturer needs to identify that the student applied an inappropriate rule (i.e., multiplying all numbers). SCK includes the ability to recognise overgeneralization and address it through error analysis and discussion of efficient strategies.

Table 3. shows the synthesis across the responses to Question 2.1.2

Student	Response	CCK Deficit	SCK Implication
S2	"said LCM"	Lacks understanding of LCM as a computed value; weak procedural knowledge	Use diagnostic questioning to probe language comprehension and task expectations
S7	"3"	Confuses LCM with GCF; lacks distinction between multiples and factors	Design instructional contrast tasks; address conceptual confusions
S14	"384"	Misapplies procedure (multiplies all numbers); lacks strategic efficiency	Use error analysis to reveal structure and efficiency in LCM computation

Using the MKT framework, it becomes evident that these student errors are not merely computational but stem from deeper conceptual misunderstandings and procedural misapplications (see Table 3). A lecturer's SCK is essential for diagnosing such errors, selecting appropriate representations, and designing interventions that move beyond rote methods to meaningful understanding. These insights suggest that professional development in content-specific pedagogical reasoning is crucial for enhancing students' mathematical proficiency, particularly in foundational number theory topics such as LCM.

Question 2.1.3 required students to write down the HCF of 12 and 18 and show their workings.

The Correct Answer and Expected Working:
Prime factorisation:

$12 = 2^2 \times 3$

$18 = 2 \times 3^2$

Common prime factors: 2 and 3

HCF = $2 \times 3 =$ **6**

Alternatively, listing method:

Factors of 12: 1, 2, 3, 4, 6, 12
Factors of 18: 1, 2, 3, 6, 9, 18
Common factors: 1, 2, 3, 6
Highest: 6

Students' responses and analysis

1. Student 2 (S2): "said HCF", S2 repeated the phrase "said HCF" rather than computing the actual value or showing steps.

   CCK analysis indicates a lack of procedural fluency and conceptual understanding of what HCF entails. The student may not understand that the HCF is a number derived from comparing factors or using prime factorisation. This confirms the research argument, which highlights that such misconceptions typically stem from gaps in learning that have accumulated over the preceding years of schooling (Benson et al., 2023).

   SCK implication: A lecturer must recognise this as a linguistic or syntactic misunderstanding, where the student has memorised a phrase but does not grasp the underlying mathematical operation. Strong SCK is required to (1) diagnose that this is not just a computational error, and (2) respond with targeted interventions, possibly rephrasing the question or using visual supports (e.g., factor trees or diagrams).

2. Student 7 (S7): Identified common factors but not the highest. S7 correctly identified common factors of 12 and 18 (e.g., 1, 2, 3, 6) but failed to select the highest among them.

   CCK analysis suggests partial conceptual understanding: the student knows how to find common factors but lacks the skill to identify the maximum from a set. The error is not in computation but in comparing values, a fundamental numerical reasoning skill.

   SCK implication: A lecturer's SCK must allow them to (1) identify this as a comparison issue rather than a content knowledge deficit, and (2) ask probing questions such as: "Which of these is the largest common factor?" or use tasks where students sort factors by size.

3. Student 27 (S27): Wrote "6" with no workings. The answer is correct, but the student did not follow the instruction to "show workings."

   CCK analysis demonstrates sound conceptual and procedural knowledge, but a potential lack of understanding of mathematical communication expectations.

   SCK implication: SCK is essential here to recognise the importance of:

   Teaching the disciplinary norms of mathematics (e.g., showing process and justification).

   Emphasising that mathematical reasoning is not only about correctness but also about making one's thinking visible.

Table 4. Illustrates the Synthesis of MKT-Informed Analysis

Student	Response	CCK Deficit	SCK Interpretation	Pedagogical Implication
S2	"said HCF"	No computational attempt; unclear what HCF is	Syntactic/language misunderstanding	Rephrase questions; connect language and math meaningfully
S7	Found common factors, missed HCF	Can compute factors, but can't select the greatest	Difficulty with comparison/selecting from a set	Use sorting and ranking exercises to highlight maximum/minimum
S27	"6" (no workings)	Strong procedural understanding	Does not recognise the value of showing work	Teach norms of mathematical communication and justification

Applying the MKT framework, particularly CCK and SCK, provides critical insights into the varying nature of student errors (see Table 4). These are not random mistakes but reflections of specific gaps in either mathematical knowledge, interpretive ability, or communication practices. Lecturers need robust, specialised content Knowledge to diagnose these subtleties and respond with appropriate strategies that go beyond remediation to cultivate deeper mathematical thinking.

This analysis underscores the need for lecturer education programmes to explicitly develop SCK, particularly in how to interpret incomplete, incorrect, or linguistically flawed student responses to standard tasks such as HCF and LCM computation.

Question 2.5 Context: Sipho and Thandi are training for a race. Sipho runs around a circular track in 18 minutes and Thandi in 24 minutes. If they start running together at 8:00 AM, after how many minutes will they be at the starting point again at the same time?

Correct Answer:

This is a Least Common Multiple (LCM) problem.
LCM (18, 24) = 72 minutes
Therefore, they will both be at the starting point again at 09:12 AM.

Students' responses and analysis

S2 and S23: "24 -18 = 6 minutes". These two students subtracted the times, assuming that the difference in completion times determines when they meet again.

CCK analysis indicates a fundamental misunderstanding of cyclical or periodic events. These students lack conceptual knowledge of synchronisation in repeated motion, a hallmark of LCM problems.

SCK implication: A lecturer must recognise this as a linear difference model, not appropriate for circular motion and periodicity. A strong SCK is needed to redirect this thinking by modelling time-based cycles visually (e.g., using number lines or clock faces).

S7: "After 24 minutes at 08:24, when Thandi arrives, Sipho will be finished". Assumes Thandi's next return is the meeting point; partially correct in observing cycles, but fails to consider the LCM of both.

CCK analysis demonstrates incomplete cyclical reasoning. S7 understands that runners return at intervals but does not compute or even conceptualise a shared return point.

SCK implication: A lecturer should interpret this as an emerging understanding of periodicity, which needs to be developed through structured examples. SCK helps lecturers guide students from individual intervals to mutual synchrony.

S11 and S41: Used HCF to get "6 minutes". They applied HCF instead of LCM, selecting the largest common factor of 18 and 24.

CCK analysis reflects confusion between GCF (used for division tasks) and LCM (used for finding the greatest common factor or for alignment or synchronisation). Indicates a lack of understanding of when to apply each concept.

SCK implication: Lecturers with strong SCK will anticipate this mix-up and teach explicit contrastive examples (1) If two events are happening, when do they occur *together* again? → LCM, and (2) What can we evenly divide into two groups? → GCF

Research suggests that teaching methods that utilize real-life settings, such as Venn diagrams and prime factorization, can enhance performance skills while also helping students establish proper connections between concepts (Caniglia & Meadows, 2018).

S14: Used (18 × 8) ÷ 24 = 6. This is a procedurally incorrect attempt at deriving a shared interval, possibly attempting to relate multiples.

CCK analysis suggests procedural trial-and-error without conceptual grounding. A student may not understand why this operation yields or doesn't yield synchronisation. This aligns with Yanga et al. (2021), who argue that the misconceptions underlying these errors often stem from a lack of relational understanding.

SCK implication: A lecturer should use this as a teachable moment to illustrate why procedures matter less than understanding the structure. SCK supports diagnosing misapplication of formulas and replacing them with reasoning strategies.

S27: Used proportions (24a = 144; 18a = 192 → 6 and 10 minutes). This student applied proportional reasoning inconsistently to try and derive a time when the runners meet.

CCK analysis shows an attempt at algebraic modelling, but with an incorrect formulation of proportional relationships. The values 144 and 192 are not clearly justified, suggesting a lack of internal consistency.

SCK implication: Lecturers need to use this as an entry point to clarify when and how proportion applies, especially in motion problems. Strong SCK supports distinguishing linear vs. periodic reasoning.

Table 5. Shows the summary of MKT-Based Error Categorisation

Student	Response	Interpretation	CCK Deficit	SCK Implications
S2 & S23	24 – 18 = 6	Misinterpreted the difference as meeting time	No concept of LCM or synchronisation	Model periodicity with visuals or manipulatives
S7	08:24 (Thandi only)	Partial understanding of cyclic motion	Incomplete concept of shared intervals	Use timelines to align cycles of motion
S11 & S41	HCF = 6	Used the wrong concept (HCF)	Confuses GCF with LCM	Contrast HCF vs. LCM using context-rich problems
S14	(18×8) ÷ 24 = 6	Arbitrary procedural approach	No justification or method sense	Emphasise structure over computation
S27	Proportion 24a = 144, etc.	Inappropriate algebraic modelling	Misused proportional reasoning	Clarify linear vs. cyclic models

The analysis of responses to Question 2.5, framed within Ball et al.'s (2008) MKT model, reveals a spectrum of misconceptions: from confusing difference with periodicity, to applying the wrong concept (HCF instead of LCM), to procedural overreliance and symbolic manipulation errors (see Tabel 5). The prevalence of these errors among pre-service lecturers indicates critical gaps in both Common Content Knowledge and Specialised Content Knowledge.

To address these, the lecturer must go beyond mathematical accuracy to emphasise (1) conceptual grounding of procedures, (2) error diagnosis strategies, and (3) instructional clarity on when and why certain mathematical tools are appropriate. This supports the development of pedagogically sound mathematics educators who are prepared not only to solve problems correctly but to understand and interpret the thinking of their future students.

Question 2.6's context: What is the greatest number of water bottles that can be evenly distributed among them if Sipho has 18 and Thandi has 24?

Correct interpretation:
This is the Greatest Common Factor (GCF/HCF) problem. It asks for the largest number of groups into which both 18 and 24 can be evenly divided (i.e., an equal number of bottles per group).

HCF (18, 24) = 6

Therefore, the greatest number of bottles that can be evenly distributed among them is 6.

Analysis of students' responses

S2 and S23: (18 + 24) ÷ 2 = 21. They added the two values and averaged them, assuming the mean of 18 and 24 reflects the "even distribution."

CCK analysis indicates a fundamental misunderstanding of the problem context and divisibility. These students misread "even distribution" as "equal share" without considering the constraint of evenly dividing both values simultaneously.

SCK implication: A lecturer must recognise this as an overgeneralization of the idea of fairness or averaging rather than a misunderstanding of number theory. Lecturers need to use SCK to highlight that "even distribution" in mathematics refers to divisibility, not mean value.

S14: No response. The absence of response may indicate a lack of confidence, complete confusion, or unfamiliarity with the concept.

CCK analysis suggests a gap in basic number theory understanding or inability to identify the mathematical operation required. This confirms the literature's position, which states that students lack mathematical knowledge and understanding due to their prior educational background, which poses challenges in the formation of robust didactic strategies (Mosvold, 2022

SCK implication: A lecturer should interpret non-responses as opportunities to assess the student's prior knowledge and emotional readiness. Strong SCK guides educators in offering low-stakes entry points for participation and identifying threshold concepts that require reteaching.

S11 and S41: Used LCM and got 72. Mistakenly used Least Common Multiple (LCM) when Greatest Common Factor (GCF) was required.

CCK analysis reflects confusion between LCM and GCF, which are often introduced together but serve different purposes. Incorrect use of LCM indicates knowledge of procedures but poor conceptual differentiation.

SCK implication: A lecturer must distinguish whether this is a procedural miscue or a conceptual mix-up and provide examples showing when and why each concept applies. For SCK, it is critical to use contextual cues (e.g., "distribute evenly" = GCF; "meet again" = LCM) to guide students.

S27: Added 18 + 24, then used "18 ÷ 42 × 100 = 42.8". Performed unrelated calculations involving proportions and percentages; possible misunderstanding of the task or misreading of numbers.

CCK analysis suggests severe conceptual confusion—no link to HCF, LCM, or logical structure of the problem. May have attempted to convert totals into percentages arbitrarily, indicating a procedural overreliance without a proper understanding.

SCK implication: A lecturer with strong SCK would recognise this as a breakdown in problem comprehension, not just calculation. SCK is necessary to design corrective feedback loops that clarify not only what the correct method is, but also why the current reasoning is inappropriate.

Table 6. Summary of the MKT-Based Categorisation of Responses

Student	Response	Interpretation	CCK Deficit	SCK Implication
S2 & S23	(18+24)/2 = 21	Averaged values instead of factoring	Misinterprets even distribution as the mean	Highlight the difference between averaging and common factors
S14	No response	No attempt due to confusion or uncertainty	Lacks confidence or understanding	Scaffold support and create low-anxiety environments
S11 & S41	LCM = 72	Applied LCM instead of HCF	Confuses LCM with GCF	Use contextual contrast tasks to distinguish them
S27	Arbitrary operations (18+24 → percentages)	Misread task or misapplied unrelated concepts	Severe procedural confusion	Reinforce problem interpretation before solution

The responses to Question 2.6 highlight a common and significant issue in mathematics education: the misinterpretation of mathematical vocabulary and context (see Table 6). Using the Mathematical Knowledge for Teaching (MKT) framework, it becomes clear that many pre-service lecturers exhibit procedural knowledge without conceptual depth (CCK issues), they are unable to recognise and respond appropriately to the underlying structure of problems (SCK deficits), and need explicit instruction in when and why to use specific mathematical tools like GCF and LCM.

Discussion

The analysis of pre-service teachers' (PSTs) understanding of HCF and LCM strongly reflects and extends the themes identified in the literature review regarding conceptual and procedural knowledge, teacher preparation, and the challenges faced in mathematics education.

Conceptual understanding and procedural knowledge

Hypothesis 1 demonstrated that PSTs possessed a strong conceptual understanding of HCF and LCM, as supported by a significant t-test result ($p < 0.001$, $d = 3.29$). This aligns with Halim et al. (2017) and Benson et al. (2023), who argue that robust conceptual understanding enables students to apply HCF and LCM correctly in problem-solving. High scores suggest that, for basic concepts, PSTs possess sufficient Common Content Knowledge (CCK) and can engage with Specialised Content Knowledge (SCK) to unpack number theory concepts, reflecting Skemp's (1976) distinction between relational and instrumental understanding.

However, analysis of specific questions (e.g., Question 2.1.2 and 2.5) reveals gaps in procedural fluency and the misapplication of concepts, which corroborates Lumadi (2014) and Mbhiza (2024), who suggest that PSTs often acquire procedural fluency without a deep conceptual grounding. For instance, PSTs who confuse LCM and HCF, or misapply procedures like multiplying all numbers for LCM, demonstrate an incomplete understanding of relationships and highlight that procedural skills alone are insufficient for accurate problem-solving (Herheim, 2023; Semper & Lizasoain, 2023).

Misconceptions impact on problem-solving

Hypothesis 2 revealed a significant negative correlation (r = –0.48) between misconceptions and accuracy in problem-solving. This supports Motilal & Fleisch (2020) and Yang et al. (2021), who highlight that misconceptions undermine students' ability to solve mathematical problems. The error analysis in Questions 2.5 and 2.6, where PSTs incorrectly used HCF instead of LCM or averaged numbers instead of finding common factors, illustrates the persistence of conceptual misunderstandings. These errors are consistent with the literature, which notes that prior knowledge gaps and misinterpretations of mathematical terminology contribute to the persistence of misconceptions (Chirinda et al., 2021; Benson et al., 2023).

From the MKT perspective, these findings reflect deficits in Horizon Content Knowledge (HCK) and Knowledge of Content and Teaching (KCT), as misconceptions limit PSTs' ability to connect content to teaching strategies and anticipate student errors (Ball et al., 2008; Chikiwa & Graven, 2023).

Strategy use and conceptual errors

Hypothesis 3 showed no significant correlation between strategy choice and conceptual errors (r = –0.12), implying that PSTs' selection of techniques (listing factors, prime factorisation, etc.) is independent of their conceptual understanding. This supports Halim et al. (2017) and Caniglia & Meadows (2018), who note that PSTs may apply correct procedures mechanically without grasping underlying concepts. This reinforces the need for teacher education programs to explicitly integrate procedural knowledge with conceptual reasoning, ensuring that PSTs understand not only how to perform calculations but also why certain methods are appropriate in specific contexts (Hiebert & Lefevre, 1986).

Specific Misconceptions and MKT Implications

Analysis of individual responses further illustrates the interplay between CCK and SCK, such as (1) Language-related misunderstandings (e.g., S2 writing "said LCM" or "said HCF") confirm Chirinda et al. (2021) on the impact of language barriers. From an SCK perspective, lecturers must distinguish between linguistic confusion and mathematical misunderstanding, designing scaffolds to address both. (2) Conceptual confusions between HCF and LCM (e.g., S7 using 3 as an answer) illustrate Halim et al.'s (2017) concern about mixing concepts due to rote learning. Strong SCK is required to design contrastive teaching strategies (e.g., real-life contexts, Venn diagrams) to clarify the differences, and (3) procedural overgeneralisation (e.g., S14 multiplying all numbers) reflects gaps in procedural fluency that align with Lumadi (2014) and Mbhiza (2024), emphasizing the need for explicit instruction linking algorithmic steps to conceptual understanding.

These findings underscore Ball et al.'s (2008) assertion that effective mathematics teaching requires not just content knowledge, but specialised content knowledge and pedagogical content knowledge to anticipate and correct errors.

Real-Life contexts and relevance

Analysis of applied problems (Questions 2.5 and 2.6) demonstrates that PSTs struggle to connect abstract concepts, such as HCF and LCM, to practical scenarios, consistent with Chikiwa & Graven (2023). Misinterpretations of "even distribution" or synchronisation of cycles highlight the necessity for context-rich instruction. Integrating real-life problems into teacher preparation can strengthen relational understanding and improve PSTs' ability to translate content knowledge into effective teaching strategies.

Implications for teacher education

The study confirms systemic gaps identified in the literature (Mosvold, 2022; Chikiwa & Graven, 2023; Saal & Graham, 2023), such as (1) strong basic conceptual understanding exists, but procedural application and context-based reasoning remain weak, (2) misconceptions directly affect problem-solving, highlighting the need for targeted interventions in teacher preparation programs, and (3) strategy use is disconnected from conceptual understanding, suggesting a need for integrated teaching approaches linking procedures, concepts, and student thinking.

The MKT framework offers a valuable lens for identifying these gaps and informing teacher education reforms. Developing both CCK and SCK is crucial to equip PSTs to diagnose student errors, use appropriate representations, and foster relational understanding, ultimately addressing the educational inequalities noted in South African mathematics education (Ngobeni et al., 2023; Wiseman & Davidson, 2021).

CONCLUSION

Overall, the findings indicate that while pre-service teachers demonstrate a strong conceptual understanding of HCF and LCM, persistent misconceptions significantly hinder their accuracy in problem-solving. Errors across tasks reflect gaps in both conceptual and procedural knowledge, revealing weaknesses in Common Content Knowledge and Specialised Content Knowledge as described in the MKT framework. These patterns suggest that the use of strategy alone does not guarantee conceptual understanding; rather, effective teaching requires the integrated development of content knowledge, diagnostic reasoning, and pedagogical decision-making skills. Strengthening the alignment between conceptual–procedural knowledge and MKT dimensions is therefore essential for preparing pre-service teachers to address PSTs' errors, interpret mathematical thinking, and teach number theory topics with greater precision and coherence.

Therefore, the findings underscore the urgent need for teacher education programmes to focus not only on strengthening mathematical content knowledge but also on developing the pedagogical tools necessary to interpret, respond to, and correct students' thinking. Equipping future teachers with this dual expertise is essential for enhancing mathematical understanding and teaching effectiveness in foundational topics such as HCF and LCM.

IMPLICATION FOR TEACHING

In instructing pre-service teachers (PSTs), lecturers must develop both conceptual understanding and procedural fluency, ensuring PSTs can compute HCFs and LCMs while accurately interpreting mathematical language and problem prompts. The analysis of PST responses demonstrates that many errors stem from conceptual misunderstandings rather than mere computational mistakes, confirming Lumadi (2014) and Mbhiza (2024), who reported that PSTs often acquire procedural skills without a deep understanding of foundational number theory concepts.

Teaching should emphasise the application of HCF and LCM in relevant problem contexts, integrating strategies such as comparison methods, incremental listing, or prime factor trees. Clear explanations of terms like “least,” “greatest,” and “maximum” are critical, particularly given the linguistic challenges identified by Chirinda et al. (2021), which can hinder conceptual comprehension. Instruction should also scaffold the interpretation of task instructions, distinguishing between averaging, distribution, and divisibility concepts, and using manipulatives to illustrate equal grouping and visual representations such as timelines or cycle charts (Halim et al., 2017; Caniglia & Meadows, 2018).

Analogies drawn from real-life contexts, such as race laps, cooking timers, or rotating gears—can model repeating events and cyclical phenomena, fostering engagement and promoting metacognitive reasoning (Chikiwa & Graven, 2023). Gap activities that highlight differences between HCF and LCM, such as event synchronisation or packaging problems, reinforce conceptual understanding prior to symbolic manipulation (Semper & Lizasoain, 2023; Yang et al., 2021). These approaches align with Skemp’s (1976) notion of relational understanding, supporting the development of a deeper mathematical understanding rather than the rote application of procedures.

Lecturers should also prioritise problem interpretation, enabling PSTs to translate scenarios into appropriate mathematical operations. Explicitly contrasting LCM and HCF applications in authentic contexts (e.g., scheduling, packaging, or distributing items) helps build both procedural and conceptual knowledge, consistent with Ball et al.’s (2008) Mathematical Knowledge for Teaching (MKT) framework, which emphasises the integration of content knowledge and pedagogical content knowledge. Classroom routines should value reasoning processes over final answers, encourage reflective thinking, and foster safe risk-taking through open-ended questioning and guided discovery.

Finally, integrating these strategies into teacher education programs can address systemic gaps identified in South African mathematics education (Mosvold, 2022; Chikiwa & Graven, 2023; Saal & Graham, 2023), including prior knowledge deficits, socio-economic disparities, and inequities in curriculum access (Ngobeni et al., 2023; Sadiki et al., 2023). Through deliberate scaffolding, contextualisation, and conceptually grounded instruction, PSTs can develop a durable understanding, mathematical literacy, and the ability to anticipate and respond to PSTs' errors, ultimately fostering inclusive and effective mathematics teaching.

RECOMMENDATIONS

The research findings have considerable implications for teacher training, particularly for pre-service teachers’ readiness to teach foundational mathematical concepts, such as HCF and LCM. This calls for the systematic design of teacher education programmes focusing on specific instructional strategies. Additionally, pre-service training programs should be more effective in teaching problem-solving skills, moving beyond procedural steps to parametric reasoning. The ability to reason from basic building blocks to complex systems is critical so that comprehensive instruction can be provided during teaching practice. Therefore, the following recommendations are suggested by this research study:

1. Teaching Focus

   The concepts of HCF and LCM should be taught to achieve a deep conceptual understanding, rather than through repetitive steps. Since a significant proportion of pre-service teachers struggle to achieve this level, instruction should be designed to include early predictive diagnosis before they begin their teaching practice. Moreover, instruction should focus on developing sophisticated problem-solving skills so that pre-service teachers are equipped to teach the pragmatic aspects of mathematics confidently. Building the connection between foundational ideas and their more complex applications should be emphasised, as it enhances one's flexibility and holistic grasp of mathematics. This is something teacher educators must directly attend to in their pedagogical planning.

2. Curriculum Development

   Meeting set goals necessitates a stronger focus on the intricate relationship between basic and advanced mathematical concepts. The curriculum must be purposefully designed to facilitate this process through systematically structured, gradual scaffolding that increases in complexity. This support enables students to develop a sense of confidence and mastery as they advance. There is also a need to provide a greater variety of challenging HCF and LCM problems grounded in real-life scenarios to better equip preservice teachers to deal with diverse classroom situations. Enhancing curriculum materials with these variations would improve the feel as their knowledge of the content deepens.

3. Assessment Strategy

   Assessments should be redefined to enable conceptual advancement and targeted, accurate identification of learning gaps. The strategic implementation of diagnostic assessments at critical stages within the teacher education framework facilitates the early detection of understanding gaps and informs targeted scaffolding strategies to be employed. Furthermore, there is a gap in the middle tier between lower-order recall and higher-order application that needs to be addressed by developing questions that target a more intermediate level, which would help build confidence and deepen understanding gradually.

   Meaningful learning can be achieved when assessments are designed to foster conceptual understanding, rather than solely focusing on procedural correctness. Such an approach would better equip pre-service teachers to address the real challenges they encounter in classroom settings.

4. Support Mechanisms

   Support structures are critical for enhancing mathematical understanding among pre-service teachers. Implementing peer-learning frameworks within pedagogical content knowledge courses can foster collaborative problem-solving and knowledge sharing. Teacher education programmes should also adopt proactive strategies to support at-risk students, including interactive tutorials, guided problem sets, and explanatory videos for challenging topics such as HCF and LCM. The observed polarisation in conceptual understanding, characterized by strong performance on basic concepts but difficulties with complex applications, underscores the need for differentiated instruction and targeted interventions to ensure equitable learning outcomes.

LIMITATIONS AND FUTURE RESEARCH

This study is limited by its focus on second-year South African pre-service teachers (PSTs) and on HCF and LCM, which restricts generalisability to other cohorts or mathematical domains. The cross-sectional design captures only a single time point, and some errors may reflect language comprehension rather than purely mathematical misunderstandings. Additionally, while reliability was reported, comprehensive content validation of the assessment instrument was limited.

Future research should expand to other foundational mathematics topics and diverse PST populations, employ longitudinal designs to track knowledge development, and investigate targeted interventions to integrate conceptual and procedural understanding. Studies could also examine the impact of language comprehension on learning and the translation of PSTs' knowledge into effective classroom teaching.

DECLARATION OF THE USE OF AI

Statement: during the preparation of this work, the author did not use AI services, except for the use of basic tools such as Google Translate and Grammarly which were used to check grammar and spelling.

DECLARATION

Author hereby declare that the research paper entitled "Common Factors, Common Struggles: Pre-Service Teachers' Conceptual Understanding of Highest Common Factors and Lowest Common Multiples in South Africa" is my original work and has not been previously submitted for any degree or examination at any other university.

All sources used or quoted have been duly acknowledged and referenced in accordance with academic conventions. I further declare that this work is a true reflection of my personal efforts and contributions, and it complies with the ethical standards required for academic research.

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